Reg.No.: | | | | | | | | |

VIVEKANANDHA COLLEGE OF ENGINEERING FOR WOMEN
[AUTONOMOUS INSTITUTION AFFILIATED TO ANNA UNIVERSITY, CHENNAI]
Elayampalayam – 637 205, Tiruchengode, Namakkal Dt., Tamil Nadu.

Question Paper Code: 2003

B.E. / B.Tech. DEGREE END-SEMESTER EXAMINATIONS – DECEMBER 2019
First Semester
Computer Science and Engineering
U19MA101 – CALCULUS
(Common to Electrical and Electronics Engineering, Electronics and Communication Engineering, Information Technology & Biotechnology)
(Regulation 2019)

Time : Three Hours   Maximum : 100 Marks

Answer ALL the questions

PART – A

(10 x 2 = 20 Marks)

1. Using Taylor's theorem, express the polynomial $2x^3 + 7x^2 + x - 6$, in powers of $(x - 1)$
2. Give an example to justify that, "the failure of differentiability implies the failure of mean value property".
3. $z = f(u)$ is a homogeneous function of x and y of degree n, and first order partial derivatives of z exist, and are continuous then prove that $xu_x + yu_y = n\frac{f(u)}{f'(u)}$.
4. If $f(x, y) = tan^{-1}(xy)$, find an linearly approximate value of $f(1.1, 0.8)$ using Taylor's series.
5. Show that the value of $\int_0^1 sin(x^2)dx$ cannot possibly be 2.
6. If f is a continuous function, find the value of the integral, $I = \int_0^a \frac{f(x)dx}{f(x) + f(a - x)}$.
7. Convert a rectangular coordinates $(-\sqrt{2}, \sqrt{2}, 1)$ into a cylindrical coordinates.
8. Find the volume of a solid generated by the revolution of the cardioid $r = a(1 + cos\theta)$ about the initial line.
9. If the two roots of an auxiliary equation with real coefficients are $3 \pm i$, then identify the corresponding homogeneous linear differential equation.

10. Why the method of variation of parameters is called so?

PART – B

(5 x 16 = 80 Marks)

11. a) i. The Taylor's series of any sufficiently differentiable function can be written as $g(x) = P_n(x) + R_n(x)$, where $P_n(x)$ is the polynomial of order n and $R_n(x)$ is the remainder term. Write the formula for $R_n(x)$ and if $g(x) = cos(x)$, then prove that $R_2(x) \geq 0$, for$|x| \leq \pi$. (2+2+2)

ii. Write Leibnitz's theorem for n^{th} derivative of the product of two functions. If $y = log\{x + \sqrt{(1+x^2)}\}^2$, then prove that $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2y_n = 0$. Hence show that $(y_{2k+1})_0 = (-1)^k \frac{(2k!)^2}{2^{2k-1}(k!)^2}$, where k is a positive integer.

(2+4+4)

(OR)

b) i. Show that among all rectangles that can be inscribed in a given circle, the square has the greatest area. (8)

ii. Let $f: \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} x & if\ x\ is\ rational \\ 1-x & if\ x\ is\ irrational \end{cases}$$

Show that f is continuous at the point a if and only if $a = \frac{1}{2}$. (8)

12. a) Find the directional derivative(s) of $f(x,y) = x^2 + y^2$at $(3,4)$ in the direction of the tangent vector to $2x^2 + y^2 = 9$ at $(2,1)$.

(OR)

b) i. If $ln(u^2 + v)$, $u = e^{x+y^2}$, $v = x + y^2$, then show that $2y\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = 0$. (8)

ii. Find the absolute maximum and minimum values of the function $f(x,y) = 3x^2 + y^2$. (8)

13. a) i. Evaluate $\int_0^1 x^{3/2}(1-x)^{3/2}dx$. (8)

ii. Find the reduction formula for $I_n = \int e^{ax} sin^n x dx$, then deduce the value of I_3 for $a = 1$. (8)

(OR)

b) Find the area of the region in the first quadrant bounded by the curves $x = 2\sqrt{y}$, $x = (y-1)^2$ and $x = 3 - y$ by integration.

14. a) i. Change the order of integration in the integral $\int_0^{\pi/2}\int_x^{\pi/2} \frac{siny}{y} dy\, dx$ and hence evaluate it. (6)

ii. Find the volume of the solid enclosed by the paraboloid $x = y^2 + z^2$ and the plane $x = 16$ by triple integration. (10)

(OR)

b) Find the volume of the given solid bounded by the cylinder $x^2 + y^2 = 1$ and the planes $y = z, x = 0, z = 0$ in the first octant by double integration.

15. a) i. Solve the given initial value problem, $y'' + 4y = g(x),\ y(0) = 1, y'(0) = 2,$

where $g(x) = \begin{cases} sinx, & 0 \leq x < \frac{\pi}{2} \\ 0, & x > \frac{\pi}{2} \end{cases}.$ (8)

ii. Find the solution of the differential equation $(D^2 + a^2)y = secax$, using method of variation of parameters.(8)

(OR)

b) Find the general solution of the differential equation $x^2y'' - 3xy' + 3y = 2x^4e^x$.
